One Negation, Two Ways of Using It:
Prasajyapratiśedha in Bhāviveka and Candrakīrti’s Argumentation

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Abstract: The famous Madhyamaka philosopher Nāgārjuna has a very special way of arguing against his opponent: he often argues and concludes that a certain thesis of his opponent should be rejected while at the same time denies that he has therefore endorsed the negation of the thesis of his opponent. This special way of argumentation had a tremendous influence upon later Indian Buddhist philosophers but has invoked two different interpretations, the Prāsaṅgika and the Svātantrika, about what exactly this special way of arguments is. While Svātantrika proposes that one should establish full hetuvidya syllogisms in accord with the ultimate truth to rebut opponents, Prāsaṅgika proposes that the only legitimate Madhyamaka way of argumentation is to deny opponents’ theses by merely indicating their absurdities. Strikingly, both Svātantrika and Prāsaṅgika emphasize that the negation used in the Madhyamaka arguments should be prasajyapratiśedha. In this paper, the authors explore how Bhāviveka and Candrakīrti, two representors of Svātantrika and Prāsaṅgika respectively, had different understandings of prasajyapratiśedha. Based on textual evidences and philosophical analysis, the authors argue that while Bhāviveka’s way of using prasajyapratiśedha in a Svātantrika argument would commit him to a certain conclusion, Candrakīrti’s way of using prasajyapratiśedha in a Prāsaṅgika argument would not have the same effect.

From a rational reconstruction, the authors then propose that a Mādhyamika, one who advocates Madhyamaka ideas, should simply reject his opponent’s thesis by drawing absurdity from it and should at the same time refrain from making any conclusion, and therefore the Prāsaṅgika interpretation is the right Nāgārjuna way of argumentation as shown in Mūlamadhyamakakārikā. In the end, based on Prāsaṅgika way of argumentation, the authors will provide a formal regimentation of Nāgārjuna’s rebuttal.

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Introduction

Prāsaṅgika and Svātantrika, two major Madhyamaka schools, advocated separately two distinct ways of rebutting an opponent, and each of them claimed that their way was the approach that Nāgārjuna used in his masterpiece Mūlamadhyamakakārikā (hereafter: MMK). Ostensibly, the main difference between the two schools is about whether a Mādhyamika should argue for and assert his/her affirmative/negative thesis either before or after repudiating his/her opponent’s thesis. While the Prāsaṅgika uses a method that we will call “prasaṅga” (simply pointing to a consequence of the opponent’s thesis that the opponent is unwilling to accept without arguing for or making a conclusion) as a special way to rebut the opponent, the Svātantrika, on the other hand, always argues for and asserts the conclusion in accord with emptiness (śūnyatā), the ultimate truth (paramārtha-satya) in the Madhyamaka school of thought. For the Prāsaṅgika, however, making any assertion about the conclusion, no matter a positive one or a negative one, would disobey Nāgārjuna’s “no-thesis” position. Because the Prāsaṅgika thinks that Nāgārjuna does not intend to propose any statement whatsoever on the level of the ultimate truth in view of the “ineffable” nature of emptiness, prasaṅga is the only

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2 According to this Prāsaṅgika/ Svātantrika distinction, Bhāviveka (500–570 CE) and his successor Sāntarakṣita (725–788 CE) represent the Svātantrika school while Buddhapālita (470–540 CE) and his successor Candrakīrti (600–650 CE) represent the Prāsaṅgika one. The Svātantrika interpretation of Madhyamaka thoughts were very influential both in China and in Tibet before the 9th century, but the Prāsaṅgikā school has gradually evolved and finally become the authoritative Madhyamaka interpretation in Tibet (not in China) since then. Moreover, modern studies and formal regimentations of Madhyamaka philosophy have largely been influenced by the Prāsaṅgikā tradition. For the historical background, please see Hsu, Bhāviveka’s Jewel, 25–34. For modern studies, please see: Garfield, The Fundamental Wisdom of the Middle Way; Napper, Dependent-Arising and Emptiness; Hopkins and Napper, Meditation on Emptiness, and Priest, “The Structure of Emptiness”, 468–480.

3 Briefly, the name of Prāsaṅgika is derived from the term prasaṅga, a method similar to the classical Reductio ad Absurdum. However, it is controversial whether the Prāsaṅgika way of argumentation, like Reductio ad Absurdum, implies a conclusion that is a negation of the original propositions. Because of this possible difference, we will not translate Prāsaṅgika as Reductio ad Absurdum in English, but just use the original Sanskrit term. For a good discussion of reductio ad absurdum or indirect proof, see Gasser, “Argumentative Aspects of Indirect Proof”, 41–49. We will have more to say about this method below and in Section 4.
adequate method used in a debate for the Prāsaṅgika on issues about the ultimate truth. Hence, for a Prāsaṅgika philosopher, the difference between the Prāsaṅgika and the Svātantrika ways of argumentation is not only methodological, but also substantial: it reveals two different understandings of Nāgārjuna’s doctrine of emptiness.4

These two ways of argumentation can be roughly characterized by the following two argumentation schemes (where “A” stands for the thesis of the opponent, “⊥” stand for a falsity or an absurdity, and deleting “A” and “¬A” indicates, respectively, the denial and the negation of A):5

<table>
<thead>
<tr>
<th>The Svātantrika</th>
<th>The Prāsaṅgika</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conclusion: ¬A</td>
<td>Assume A;</td>
</tr>
<tr>
<td>Reasons</td>
<td>Derive ⊥;</td>
</tr>
<tr>
<td>Examples</td>
<td>¬A (also ¬A).</td>
</tr>
</tbody>
</table>

From the schemes, one can clearly see that the Svātantrika way of argumentation meets Dignāga’s basic criteria for a valid hetuvidya syllogism: the thesis to be proved (siddhānta, pratijñā), reason (hetu), and example (dṛṣṭānta). Moreover, the thesis has the subject (pakṣa) and the target property to be proved (sādhya-dharma). The main purpose of hetuvidya syllogism is to prove that the subject in the thesis does (or does not) possess the target (possibly negative) property by the tight connection between the reason and the property. Note that it is a requirement of Dignāga’s hetuvidya syllogism that the arguer must explicitly assert the conclusion, so Svātantrika is actually proposing that a Mādyamika should always assert his/her conclusion when s/he argues with his/her opponents. This trait will be

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4 Many later Indo-Tibetan scholars, such as Tsong Kha Pa, endorse Candrakīrti’s critiques on the Svātantrika and takes the Prāsaṅgika to be the highest and the correct Madhyamaka school. See: David Ruegg, The Buddhist Philosophy of the Middle, 4. In contrast to Tsong Kha Pa, some Tibetan Buddhist scholars, such as Go Rams Pa (1429–1489 CE) and Bu Ston Rin Chen Grub (1290–1364 CE), argue that the differences between the Prāsaṅgika and the Svātantrika are merely methodological and insubstantial; see: Mi Pham, L'opalescent Joyau, 6. Debates over the question on whether the difference between the Prāsaṅgika and the Svātantrika is merely methodological but not substantial continues between modern Buddhist scholars, including Ames, Huntington, and McClintock. According to Dreyfus’s report, Ames and McClintock reply to the question with a positive answer, while Huntington gives a negative reply. No consensus about this question has been reached so far. See: Dreyfus and McClintock, Svatantrika-Prasangika Distinction, 6.

5 The exact meaning of the deletion will be explained in Section 4.
crucial for our later discussion. On the contrary, as mentioned above, the Prāsaṅgika way of argumentation only rebuts the opponents’ theses by indicating the absurdities of the theses without making any assertion.

Intriguingly, both the Svātantrika and the Prāsaṅgika, represented in this paper by Bhāviveka and Candrakīrti respectively, claim that the negation used in their arguments/rebuttal is prasajyapraṭiṣedha, usually translated as non-implicative negation, which syntactically is construed with a verb which negates the action or purported fact without further implication. However, even if they adopt the same term “prasajyapraṭiṣedha” for the type of negation that should be used in Madhyamaka arguments/rebuttal, one plausible question to be asked would be: does prasajyapraṭiṣedha used in both schools have the same connotation? If the negations used in Svātantrika and Prāsaṅgika arguments are the same prasajyapraṭiṣedha, do these two schools just arrive at the same end by different means? After all, both schools eventually apply the same non-implicative negation to the opponents’ theses. On the other hand, if prasajyapraṭiṣedha has different meanings in these two schools, how should we clarify the air of this discrepancy? In the following sections, by examining the textual evidences provided by Bhāviveka and Candrakīrti and their models of argumentation, we will argue that the negations used in the two schools are actually different: while the prasajyapraṭiṣedha used in Prāsaṅgika is a “non-committal” negation, in Svātantrika it is a non-implicative negation. Moreover, by our rational reconstruction, we suggest that Prāsaṅgika’s interpretation is more faithful to Nāgārjuna. In the final section, we will review previous logical formalization of Nāgārjuna's philosophy and provide a formal regimentation based on our understanding of prasajyapraṭiṣedha in Prāsaṅgika.

1. Two Negations and Bhāviveka’s Way of Argumentation

In the Indian tradition, grammarians often distinguish two usages of negation: prasajyapraṭiṣedha and paryudāsapraṭiṣedha (implicative negation). Syntactically, this is a distinction about to which part of a sentence the negative word is attached. Prasajyapraṭiṣedha denotes a negation which refers to the predicate or the verb of a

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6 While prasajyapraṭiṣedha is usually translated as “non-implicative negation”, paryudāsapraṭiṣedha is “implicative negation”. However, since we argue that prasajyapraṭiṣedha is used differently in Svātantrika and in Prāsaṅgika, we keep prasajyapraṭiṣedha untranslated in this section, and adopt non-implicative negation for paryudāsapraṭiṣedha.
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statement (e.g.: “something is not X”), while the implicative negation designates a
negation referring to another word that combines with it to form the predicate (e.g.: “something is a non-X”). This syntactic difference corresponds also to a semantic
difference according to Bhāviveka:

When one says “[There is] no white silk fabric”, one is only to negate the
white silk fabric, but without any power to imply black, red, or yellow silk fabric.8

This example by Bhāviveka shows how people may be confused about implicative
negation and prasajya-pratiṣedha. From the semantic point of view, the implicative
negation implies other possible affirmations of a positive sentence, while the
prasajya-pratiṣedha simply negates the entire sentence without implying any such
possible affirmation, as used in the example.

Given this distinction, Bhāviveka proposes that when a Mādhyamika, such as
Nāgārjuna, uses a negation to rebut a proposition of his/her opponent from the
perspective of the ultimate truth, that negation s/he utters must be the
prasajya-pratiṣedha.9 For example, in his commentary on Nāgārjuna’s MMK,
Prajñāpradīpa, Bhāviveka proposes that the negation used in MMK Chap.1 v.1 to
reject all the possibilities of the arising of things should be understood as
prasajya-pratiṣedha. Thus, Bhāviveka states:

Moreover, [when Nāgārjuna says that a thing is] not produced from itself, he
just means [that it is] not produced by itself [without any affirmative
implication]. If [one has a] different understanding and says that it is not
produced by itself [but by other things], that understanding is not correct,
since that would imply that it is produced by other things.10

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7 For example, in Āpadeva's Mīmāṃsāyāyaprakāśa 330f: “Where the negation particle is
connected with the word that follows it, this is to be understood as an implicative negation
(paryudāsa). While the negation particles are associated with an activity it is to be
understood as [prasajya-] pratiṣedha”.
8《大乘掌珍論》, CBETA, T30, 270c12–15
10《般若燈論》, CBETA, T30, 52b26–28
Here, Bhāviveka’s point is clear: if the negation used to rebut a thesis in MMK is understood as other than the *prasajyapratisedha*, i.e. as the implicative negation, Nāgārjuna’s verse would imply that a thing is produced by other things. This last implication, however, is not what Nāgārjuna or a Mādhyamika would want to say from the perspective of the ultimate truth, because, for Nāgārjuna in particular and for the Madhyamaka school in general, a thing is neither produced by itself nor by others from that perspective; the production of a thing is simply empty from the perspective of the ultimate truth.

Having this distinction in mind, readers are now in a position to understand Bhāviveka’s own way of argumentation, which can be exemplified by the beginning verse of *Ta-sheng chang chen lun* (大乘掌珍論):

> In the ultimate level, conditioned things are empty, because they are produced from conditions, like a magical production. The unconditioned is not real because it is not produced, like a sky-flower.\(^\text{11}\)

This verse represents the main thesis of *Ta-sheng chang chen lun*, and the rest of the text contains the opponent’s counter arguments and Bhāviveka’s counter-counter arguments. Each sentence of the verse actually comprises an argument, thus this verse contains two arguments, the first of which is as follows (the second one can be spelled out in a similar way):

**The First Argument:**

**Hidden Major Premise:**

_Ultimately, things that are produced from conditions are empty._

**Minor Premise:** _Conditioned things are produced from conditions._

**Conclusion:** _Ultimately, conditioned things are empty._

**Similar Example:** _Magical production._

In the beginning of this verse, Bhāviveka uses ‘in the ultimate level’ or ‘ultimately’ (paramārthataḥ) to make the opponent understand that the two arguments in this

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11 Here we use Ruegg’s translation with some adjustment. Originally, Ruegg uses “in reality” to translate the Sanskrit term “paramārthataḥ”; however, since there are two levels of truth in the Madhyamaka school, i.e. the conventional truth and the ultimate truth, here we use ‘in the ultimate level’ instead of ‘in reality’ for the sake of clarity. For the original translation, please see: Ruegg, *The Literature*, 63. For the original Chinese verse, see 《大乘掌珍論》, CBETA, T30, 268b21–22.
verse are restricted to the perspective of the ineffable ultimate truth, not to that of the conventional truth which is how things appear to us and what we take for granted. The subject, the target property to be proved, and the reason in the first argument are, separately, “conditioned things”, “empty”, and “produced from conditions”, with “magical production” as a similar example. One can clearly see that the structure of this argument meets Dignāga’s criteria for a valid hetuvidya syllogism. Similarly, the second argument also contains the subject (unconditioned things), target property to be proved (not real), the reason (not produced) and the similar example (sky-flower).

It is clear from the above example that Bhāviveka adopts Dignāga’s hetuvidya syllogism to establish a Madhyamaka’s thesis about the ultimate truth, and, actually, he is the first one who synthesizes the Mādhavaka thoughts and Indian classical logic (hetuvidyā) to rebut the opponent. Note that it is a requirement of Dignāga’s hetuvidya syllogism that the arguer must explicitly assert the conclusion, so Bhāviveka is actually proposing that a Mādhyamika should always assert his/her conclusion when s/he argue with his/her opponent. This trait will be crucial for our discussion in section 3. Although prasajyapratishedha in Svātantrika arguments can indeed prevent the implication of other affirmations, in this sense non-implicative, it cannot prevent the affirmation of the conclusion itself, given the nature of hetuvidya syllogism. In sum, for Bhāviveka, a Mādhyamika should not merely use prasāṅga to rebut his/her opponent, s/he should also assert his/her own

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12 In Dignāga’s logic system, a hetuvidya syllogism is something like an enthymeme syllogism and should be created with three parts: the thesis, the reason, and a similar example and/or a dissimilar example. Most importantly, the thesis (conclusion) of a hetuvidya syllogism must always be asserted by the arguer. Also of note, in the case of this opening verse, there is no dissimilar example that can be offered, because, for a Mādhyamika, everything is empty. For more information about Dignāga’s logic system, please see Hetuvidyā nyāya dvāra śāstra (in Chinese: 《因明正理門論》).

13 In Bhāviveka’s truth system, he proposes that there are two kinds of ultimate truths: unverbalizable ultimate truth (aparyāya paramārtha) and concordance ultimate truth (paryāya paramārtha) which is the verbal and conceptual representations of the ultimate truth. According to this classification, the conclusions of Svātantrika arguments belong to concordance ultimate truth which can guide people to attain the unverbalizable ultimate truth. Therefore, despite the ontological status of concordant ultimate truth, the usage of prasajyapratisedha in Svātantrika cannot dispel all linguistic statements or judgements, but it at least derives the conclusion at the level of the concordant ultimate truth. However, due to limited space, we are unable to provide more discussion on this special classification by Bhāviveka. For more discussion, please see: Lusthaus, Buddhist phenomenology, 449; Ruegg, The Buddhist Philosophy of the Middle, 170.
thesis in accord with the ultimate truth.\textsuperscript{14} Moreover, the negation used in a Mādhyamika conclusion should be understood as the \textit{prasajyapratīṣedha}; other interpretations would be incorrect.

2. Candrakīrti’s Way of Argumentation

In \textit{Prasannapadā Mūlamadhyamaka-vṛti} (hereafter: \textit{PPMV}) Candrakīrti harshly scolds Svātantrika Mādhyamika, such as Bhāviveka and his disciples, as just logicians obsessed with making arguments but not true Mādhyamika at all.\textsuperscript{15} Candrakīrti then argues that Mādhyamika should not adopt the Svātantrika’s way of arguing because Mādhyamika should have “no view” about the ultimate truth.\textsuperscript{16} Due to this tenet, Candrakīrti therefore thinks that the Madhyamaka way of arguing for the conclusion by offering an independent argument violates the core spirit of the Nāgārjuna. For Candrakīrti, this idea echoes the 29th verse in \textit{Vigrahavyāvartanī} (in Chinese: 《迴諍論》), where Nāgārjuna affirms:

\begin{quote}
If I had some thesis, the defect [just mentioned] would as a consequence attach to me. But I have no thesis, so this defect is not applicable to me.\textsuperscript{17}
\end{quote}

Following this underlying principle, Candrakīrti proposes that:

\begin{quote}
For those who establish the independence inference (argument), the mistake will be on their side. We do not establish the independence inference (argument), since the function of argument is merely to rebut the opponent.\textsuperscript{18}
\end{quote}

In the quoted passage, Candrakīrti indicates that the correct way of making a Madhyamaka argument about the ultimate reality is \textit{merely} to refute the opponent by \textit{prasaṅga}, as Buddhapālita (and Nāgārjuna) always does.

Candrakīrti also criticizes Bhāviveka that, if he really accepts the Madhayamaka doctrine that everything is ultimately empty, then it is impossible for

\begin{footnotes}
\item[16] Candrakīrti, \textit{PPMV}, 16.
\item[18] Candrakīrti, \textit{PPMV}, 34.
\end{footnotes}
him and his opponent to have the same understanding about the subject term in the disputed thesis because only Mādhyamika, not his opponent, would take it to be ultimately empty. And, according to the Indian hetuvidya tradition, there can be no argument unless the speaker and his rival have consensus on the meaning of the subject term in the argument. Consequently, Bhāviveka’s way of argumentation will violate this basic principle of hetuvidya logic. Moreover, if Bhāviveka agrees that everything, including the subject of the thesis, property to be proved, the reasons, and the similar/ dissimilar cases, is ultimately empty, how could he then make a valid argument that contains a subject, explanandum, etc., which are all empty? If Bhāviveka does accept that everything is ultimately empty, how can he establish a thesis with a subject term that refers to things that do not exist at all? That is to say, if the Mādhyamika uses Dignāga’s Indian syllogism to establish his own thesis, he would have to commit to the existence of every component in the thesis, and that will be inconsistent with the Madhyamaka main doctrine that everything is empty *ultimately*. Therefore, Candrakīrti firmly rejects the Svātantrika way of argumentation and proposes that the only suitable way of argumentation that fits the Madhyamaka is *prasāṅga*.

Nevertheless, it is striking that Candrakīrti also interpreted the negation used in *MMK* and Prāsaṅgika inference to be *prasajyapraṭiśedha*:

> Does not “[a thing is] not produced from itself” establish the undesirable [thesis] “from others”? [answer:] It is not the case, since the *prasajyapraṭiśedha* is what wants to be said, and therefore “from others” will be dispelled as well.19

It should be noticed that here “a thing is not produced from itself” is not a conclusion derived from reasons and examples as in the argument by Svātantrika, but just a *prasāṅga* refutation of the opponents’ thesis that “a thing is not produced from itself”. Therefore, for those Mādhyamika making svātantrika inferences, such as Bhāviveka, they have to accept the conclusion derived from the inferences in accord with ultimate truth, and therefore the *prasajyapraṭiśedha* used in the inferences will still imply the assertion the conclusion. By contrast, Candrakīrti had no intention of making any assertion when rebutting the opponents.

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19 For original Sanskrit, please see: PPMVS on MMK 1-1:

> nanu ca naiva svata utpanṇā ity avadḥārayamāṇe parata utpanṇā ityaniṣṭam prāpnoti | na prāpnoti, prasajyapraṭiśedhasya vivakṣitavat parato 'pyutpādasya pratiśetsyamāṇatvāt |.
the prasajyapratiṣedha in prāsaṅgika as merely the denial of the opponents’ thesis. Therefore, according to those textual evidence and analysis, we name the prasajyapratiṣedha in Svātantrika “non-implicative negation”, and the prasajyapratiṣedha in Prāsaṅgika “non-committal negation”. This discrepancy, which will be further explored in the following section, is crucial for our rational reconstruction of Nāgārjuna’s philosophy.

3. The More Plausible Interpretation

Both the Prāsaṅgika and the Svātantrika ways of arguing intend to interpret how Nāgārjuna rebuts his opponent’s thesis in MMK. The main difference between the two interpretations is whether Nāgārjuna asserts his conclusion or his own thesis after he rejects his opponent’s thesis by drawing falsity or absurdity from the latter. It seems at first sight that the Svātantrika’s interpretation must be right; after all, Nāgārjuna apparently makes many assertions, though all of them being negative, after drawing falsity or absurdity from his opponent’s theses or assumptions. Especially, Nāgārjuna apparently asserts that a thing cannot be self-caused (or that it cannot be caused by other things, caused by both, or uncaused) by drawing absurd consequences from the assumption that it is self-caused (or the assumption that it is caused by other things, caused both by itself and other things, or that it has no cause), and, he apparently aims to assert that nothing has an intrinsic nature, which is the negation of his opponent’s thesis that at least something, i.e., the production of a thing, has an intrinsic nature. However, closer inspection of Nāgārjuna’s two-truth theory shows that facts are not as they appear.

After all, Nāgārjuna is trying to see the whole debate, as Bhāviveka points out, from the perspective of the ultimate truth, and, as mentioned in section 1, everything whatsoever, according to Nāgārjuna, is empty from this ultimate perspective. Yet, if everything is ultimately empty and therefore ultimately does not exist, what we say about anything is ultimately about nothing and therefore cannot be true from this ultimate perspective. Conversely, if there is any “truth” from this ultimate perspective, it must be something ineffable. Thus, any assertion about the ultimate truth, no matter a positive or a negative one, must not be true (and must not be false either, as we shall see below) ultimately. How then can Nāgārjuna assert, from this ultimate perspective, any truth about what is ultimately empty if he is faithful to his own two-truth theory? It seems that Nāgārjuna must not be making
any assertion, either a positive or negative one, when he rebuts his opponent’s thesis after drawing falsity or absurdity from the thesis and wants to convey something “true” from this ultimate perspective. That he apparently asserts any statement must only be an appearance.\(^{20}\)

One can also view things from another angle. Were Nāgārjuna to assert, say, “a thing cannot be self-caused” after he draws the absurdity from the assumption that the things are self-caused, he would be forced to accept that one of the other three possibilities mentioned in the verse (that it is caused by other things, caused by both, or uncaused) must be true, for these possibilities jointly exhaust all possibilities in which a thing is not self-caused. Nāgārjuna should not go on to deny all of them. The fact that Nāgārjuna goes on to reject all possibilities shows that he is not, contrary to appearance, asserting the negative sentence “a thing cannot be self-caused” after he draws out the absurdity of the assumption.

Several related issues follow: if Nāgārjuna does not mean to assert those negative sentences that he utters after reducing his opponent’s theses to absurdity in MMK, how should we understand the function of those negative sentences in MMK? Especially, how should we understand the function of the negative word “not” used in these utterances? Should we understand it as the prasajyapratiśedha that Bhāviveka suggests? If not, should we understand Nāgārjuna’s apparent assertion of these negative sentences as a “pretension”? We suggest that “no” is the answer to the final two questions. After all, what is the benefit of pretending to make a negative assertion when one’s aim is to convey some “ineffable truth” about the ultimate reality?\(^{21}\) And, after all, using the word “not” in the Bhāviveka’s prasajyapratiśedha, which we call the non-implicative negation, is still making an assertion of a negative sentence with the non-implicative “not”\(^{22}\) and we have seen

\(^{20}\) Or it could be a mistake. However, we do not think that it is plausible to attribute such a mistake to Nāgārjuna; here, we adhere to the famous principle of charity proposed by D. Davidson. See: Davidson, *Inquiries into Truth*, Chapter 13.

\(^{21}\) What is an ineffable truth? It is about the ultimate reality but cannot be expressed in any language. Since it is not expressible in any language, it is perhaps misleading to call it a “truth”. However, if one takes a truth to be a mind-independent proposition and allows that there are propositions that cannot be expressed in any language, then it makes perfect sense to talk about an ineffable truth. Does Nāgārjuna intend to convey some such ineffable truth to his readers of MMK? On one reading, especially the one hinted by later Chinese Chan philosophers, Nāgārjuna does intend to convey some such ineffable truth to his readers. Is this a possible mission? If so, how? These are questions that we need not answer here (though we think that they can be answered), for our purpose here is merely to point out a way to understand Nāgārjuna’s rebuttal, not to defend such a possibility.

\(^{22}\) For more on this claim, please see the final paragraph of this section.
that this way of understanding *MMK* will wrongly attribute the attitude of disobedience to his own two-truth theory to Nāgārjuna. But, then, to repeat, how should we understand the function of those negative sentences found in *MMK*? Especially, how should we understand the function of the negative word “not” used in these negative utterances?

We believe that the word “not”, when uttered by Nāgārjuna in *MMK* to rebut his opponent’s thesis, is used, neither as Bhāviveka’s non-implicative negation nor as implicative negation, but in a third way as Candrakīrti’s prasajyapratiśedha, which we call “the non-committal negation”, referred to by the *deletion* of a sentence mentioned in the introduction. Unlike the non-implicative use of “not”, which commits one to the truth of a negative sentence containing a “not” with a wider scope, and unlike the implicative use of “not”, which commits one to the truth of a negative sentence containing a “not” with a narrower scope, the non-committal use of the word “not” commits one to neither, but *merely indicates* a speech act of denial or a propositional attitude of rejection to a sentence or a proposition.²³ We emphasize the phrase “merely indicate” because when one uses the word “not” implicatively or non-implicatively, it is used, of course, to deny (or reject) a sentence or a proposition; however, one typically does more than that: s/he also asserts (or accepts) a negative sentence or a negative proposition. The non-committal use of the word “not”, by contrast, *merely indicates* a speech act of denial (or a rejection) of a sentence or a proposition *and no more*. Therefore, the denial (or the rejection) of a sentence *p*, indicated by such non-committal use of “not”, should not be thought of as accompanied by (or as implying or containing) an assertion (or an acceptance) of the negation (either implicative or non-implicative) of *p*. One does not have to assert (or accept) not-"p" in order to *merely* deny *p* (or *merely* reject *p*), and in some cases (especially when one takes that both *p* and not-*p* are somehow “defective”), one can both rationally deny (and/or reject) *p* and deny (and/or reject) not-"p".²⁴

²³ A speech act, also called an “ilocutionary act”, is what one does with his/her utterance (Austin, *How to Do Things With Words*, 98), while a propositional attitude is one’s mental attitude toward a proposition. Speech acts are not propositional attitudes and vice versa. Because they are different, our explanation is actually a disjunctive one: either the word “not” merely indicates a speech act, or it merely indicates a propositional attitude of Nāgārjuna. We do not know which disjunct of the disjunction is true and we do not exclude the possibility that both disjuncts are true either. However, either disjunct is enough to lend its support to our conclusions.

²⁴ For a good discussion of negation, denial, and speech acts, see J. Moeschler, “The Pragmatic Aspects”, 51–76; Ripley, “Negation, Denial, and Rejection”, 622-629; and Horn
To support our claims, we cite here a discussion made by H. Field of the distinction between rejecting a proposition \( p \) and accepting its negation \( \neg p \) (similar things can be said about the distinction between the denial of \( p \) and the assertion of \( \neg p \)):\(^{25}\)

[One] can . . . distinguish between rejection [of \( p \)] and acceptance of the negation [of \( p \)]. Rejection should be taken to involve, at the very least, a commitment not to accept [\( p \)] . . . [A] defender of [this view] should . . . take rejection not to require acceptance of the negation. By doing so, we can allow for the simultaneous rejection of both a sentence and its negation; even the full rejection of both.

The point here is that one should not confuse the rejection of a proposition \( p \) with the acceptance of its negation \( \neg p \) (implicative or non-implicative); especially, rejecting a proposition \( p \) does not necessarily imply accepting \( \neg p \) (implicative or non-implicative). Similarly, one should not confuse the denial to a sentence \( p \) with the assertion of its negation \( \neg p \) (implicative or non-implicative); especially, denying a sentence \( p \) does not necessarily imply asserting \( \neg p \) (implicative or non-implicative).

Unfortunately, even though the speech act of merely denying \( p \) (or the propositional attitude of merely rejecting \( p \)) should be distinguished from the speech act asserting \( \neg p \) (or the propositional attitude of accepting \( \neg p \)), the most ordinary means that we have for merely denying a sentence (or for merely rejecting it) is still to use the word “not”, which may be the main source of confusion.\(^{26}\) If one asserts that “the round square is round” and we intend merely to deny it (without committing to its negation) because we think that there is no such a thing as the round square, it is very natural for us to respond to such assertion with “No, it is not!” Of course, this may mislead one to believe that part of our what we mean is that “it is not round” (where “not” is used non-implicatively) or “it is non-round” (where ‘non’ is used implicatively), but it should be clear from our explanation that, since we believe that there is no such thing as the round square, we do not intend to assert

\(^{25}\) Field, *Saving Truth from Paradox*, 73–7

\(^{26}\) We can avoid this confusion by, say, stipulating that, whenever one utters a sentence with his/her nose being pulled out by his/her fingers or with his/her head shaking violently, s/he is merely denying it without asserting any negative sentence. However, this is not the convention that we currently have.
or accept either one of them. If one goes on to ask us “so it is not round?” or “so it is non-round?”, we will certainly reply to both questions by saying “neither!” out loud. Our reply, to be sure, presupposes a Fregean view about “defective sentences” with empty names, according to which a sentence of whatsoever kind is not true (and not false either) if its subject term does not refer, but we think that this view makes perfect sense when applied to sentences about the realm of emptiness, i.e., about the realm of ineffable things.27

That being said, we can now go back to see how Nāgārjuna uses the word “not” in MMK to rebut his opponent’s thesis and why he uses it in that way. When Nāgārjuna takes a state-of-affair (or a proposition, if you like) \( p \) to be ineffable from the ultimate perspective, it is also reasonable for him to take, just like a Fregean would do, its negation not-\( p \) to be ineffable from the same perspective. As a result, if Nāgārjuna is faithful to his doctrine that everything is ultimately empty and ineffable from the ultimate perspective, he simply cannot be asserting “not-\( p \)” even if he utters it after concluding, by reasoning, that his opponent’s claim of \( p \) leads to absurdity. Therefore, the “not” used by Nāgārjuna in MMK actually indicates neither the assertion of a negative sentence nor the acceptance of a negative proposition, but merely the denial of the thesis at issue. It is only because our ordinary means to merely deny a sentence (or to merely reject it) is still to use the word “not”, the Svātantrika is misled into believing that Nāgārjuna is actually making an (negative) assertion after he draws the absurd consequences of his opponent’s thesis. But this is a confusion. In short, the Prāsaṅgika is right: whenever the Mādhyamika successfully reduces his/her opponent’s thesis to absurdity, s/he should simply reject or deny his/her opponent’s thesis without making his/her own negative conclusion if s/he is to see things from the ultimate perspective.

It may be suggested that Bhāviveka’s implicative/non-implicative distinction is nothing but our mere-denial/assertion or mere-rejection/acceptance distinction and therefore the dispute between the Svātantrika and the Prāsaṅgika is nothing but verbal. This suggestion, however, is not plausible. After all, the Svātantrika is trying to argue in the hetuvidya way that Dignāga’s demands, and an assertion of the conclusion is always needed in a hetuvidya argument. If the conclusion is a negative sentence, no matter whether the “not” is used implicatively or non-implicatively, an assertion is still made, which makes the conclusion a mere denial impossible.28

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27 For details of this Fregean view about defective sentences with empty names, see: Gottlob Frege, “On Sense and Reference”, 25–50.
28 It may be argued that non-implicative negation lacks assertoric force, and, if so, how
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Besides, an implicative or a non-implicative negation is always, when uttered, a part of a sentence or a part of a proposition, but the mere denial of a sentence or the mere rejection of a proposition is an act or a mental attitude which can never be a part of a sentence or a proposition. Thus, it is nonsensical to equate Bhāviveka’s implicative/non-implicative negation distinction with our mere-denial/assertion or mere-rejection/acceptance distinction.

4. A Formal Regimentation of Nāgārjuna’s Rebuttal

We have concluded that the Prāsaṅgika is the right interpretation of how Nāgārjuna rebuts his opponent in MMK: whenever Nāgārjuna successfully reduces his opponent’s thesis to absurdity, he simply rejects or denies his opponent’s thesis without making his own conclusion. This also explains why Nāgārjuna claims that he has not proposed any thesis at all. It remains to be explained whether this way of argumentation can make perfect sense from a logical point of view. There are a few contemporary philosophers, especially Garfield, Priest, and Westerhoff, who have addressed this issue. We will briefly review their views before we give our own.

Like Bhāviveka, Westerhoff also distinguishes two kinds of negation and calls them “presupposition-preserving” and “presupposition-canceling” respectively. It is difficult to tell whether Westerhoff’s distinction is exactly the same as Bhāviveka’s implicative/non-implicative distinction, because there is evidence does its use by Bhāviveka commit him to making an assertion about a negative sentence? But this is exactly what we have been trying to argue in the paper: a non-implicative negation, especially being the conclusion of a hetuvidya argument, still has some assertoric force; it commits one to the acceptance of a negative sentence. The difference between the implicative and the non-implicative negation is not that the former has, while the latter lacks, an assertoric force. The distinction is rather that the former has, while the latter lacks, the implication that some other positive affirmation is true. The utterance of a non-implicative negation still commits one to accept a certain negative sentence, thus still having some assertoric force. By contrast, what we call the “non-committal” use of the word “not” does not commit one to any such positive or negative sentence at all.

You may think that, even if an act or mental attitude cannot be a part of a sentence, still it is something that can be truly asserted or correctly described by a true sentence. Our reply to this criticism is this: that is certainly true, but only from the conventional perspective. Viewed from the ultimate perspective, such a description or such an assertion should also be denied or rejected.

indicating both that he intends them to be identical\(^{31}\) and opposite\(^{32}\). Fortunately, we do not have to get involved into this interpretational problem; we can simply say two things about Westerhoff’s interpretation: (a) as long as he takes Nāgārjuna’s rebuttal to be an assertion with a non-implicative or a presupposition-canceling negation, he is wrong about the rebuttal for reasons mentioned in the previous section; (b) on the other hand, if he takes Nāgārjuna’s rebuttal to be something like a mere illocutionary act,\(^{33}\) he is right about the rebuttal and agrees with us. The only issue left for (b) is whether Nāgārjuna’s way of argumentation can make perfect sense from a logical point of view, which is a topic Westerhoff does not discuss.

More Recently, Garfield & Priest developed a novel but controversial interpretation of how Nāgārjuna rebuts his opponent in *MMK*.\(^{34}\) They take a hint from Westerhoff and suggest that the rebuttal of a thesis in *MMK* by Nāgārjuna should be taken as “the external negation” which is another name that Westerhoff uses for his presupposition-canceling negation. The reference to Westerhoff’s external negation also creates ambiguity between (a) and (b) in the above paragraph, and we will comment similarly as we did for Westerhoff’s suggestion. So far, there is nothing very exciting in their interpretation. However, their formal regimentation of Nāgārjuna’s way of rebuttal has a few very interesting features that we will explain and comment on in the rest of this section.

Garfield and Priest’s formal interpretation of how Nāgārjuna rebuts his opponent in *MMK* consists mainly of two parts: a part about the positive *catuskoti* and a second part about the negative *catuskoti*.\(^{35}\) In the first part, they take Nāgārjuna to be a logician who thinks that all sentences about the conventional

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\(^{31}\) Especially, he identifies his presupposition-preserving/presupposition-canceling distinction with Bhāviveka’s prasajyapratiśedha and paryudāsapratīṣedha.

\(^{32}\) Especially, he suggests that his presupposition-canceling negation is an illocutionary act.

\(^{33}\) Cotnoir in 2015 also suggests that we take Nāgārjuna’s rebuttal along the line of (b) here.

\(^{34}\) Not everything they say in that article is relevant to the current issue; especially, the main focus of their joint paper, namely, the logical form of a *catuskoti* (either a positive one or a negative one) is not our current concern.

\(^{35}\) ‘Catuskoti’ means ‘four corners’ in Sanskrit. Nāgārjuna (as well as many other ancient Indian philosophers) often divided possibilities about a thing into four kinds and then reasoned about whether any one of them would hold. When he divided possibilities of a thing in this way, we call the four kinds of possibilities a ‘catuskoti’ (or four koṭi). One example of such a catuskoṭi (self-caused, caused by other things, both, and no cause) can be found in verse 1.1 of *MMK* mentioned in section 1 of this paper. When Nāgārjunas affirms, usually from the conventional perspective, that at least one of a four koṭi holds, such catuskoṭi is said to be a ‘positive catuskoṭi’. Nāgārjuna, however, often denies all four koṭi from the ultimate perspective; in this case, it is said to be a ‘negative catuskoṭi’.
reality can be divided into four mutually exclusive non-empty categories: true-but-not-false (t), false-but-not-true (f), both-true-and-false (b), and neither-true-nor-false (n). This part of their interpretation is rather controversial and we will reject it for reasons discussed in the next paragraph. On the other hand, we think that their interpretation of the negative catuṣkoṭi is closer to truth: they take Nāgārjuna to be a logician who adds one more value, e (standing for “ineffable”), to the original values for the conventional truth when using a negative catuṣkoṭi. However, their interpretation of the negative catuṣkoṭi are implausible for two reasons: (a) since a sentence is certainly something effable, it makes no sense to attribute the value e (“ineffable”) to it; (b) the value e should not be an extra value that can be attributed to sentences with conventional values; it is rather, according to Nāgārjuna, the value that everything (including every sentence) ultimately has from the ultimate perspective. These two problems, however, are nicely corrected by Priest (2018) in the following ways: (1) the value-bearers in the negative catuṣkoṭi are now assumed to be state-of-affairs rather than sentences; (2) each state-of-affairs is now assumed to have the value e and possibly an extra conventional value. Priest calls this final version of the formal semantics for MMK “plurivalent’ semantics, which is quite inspiring in interpreting Nāgārjuna’s rebuttal, though we still believe that their four-valued construal of the positive catuṣkoṭi is quite wrong.36

There are two main reasons for why we believe that the four-valued construal of the positive catuṣkoṭi is quite wrong. To spell them out in full detail, however, would require another paper, so our comments below will only be brief. First, there is a problem of literature and historical support for their interpretation. By making both-true-and-false (b) a non-empty category for a positive catuskoti, Garfield and Priest are actually interpreting Nāgārjuna's view about the conventional reality as a sort of dialetheism, according to which there are true contradictions in the conventional reality. But it seems to us that such a dialetheist interpretation of Nāgārjuna's view of the conventional reality has neither literature nor historical support. Second, none of the formal systems (FDE, FDEe, and P-FDEe) that they propose seems to be adequate to ground the logic at play in MMK for the simple reason that they are all too weak. As Cotnoir (2015) points out, Nāgārjuna often uses classically valid inference patterns, such as Modus Ponens, Modus Tollens, Hypothetical Syllogism, reductio ad absurdum (RAA) to argue against his opponent in MMK, but none of these inference patterns are valid in any of their proposed logic systems.

36 See: Priest, The Fifth Corner of Four.
Due to the above observations, we therefore suggest the following formal treatment of Nāgārjuna’s rebuttal in MMK. In Nāgārjuna’s conception, each state-of-affairs can be viewed from two perspectives, a conventional perspective and an ultimate perspective. Viewed from the ultimate perspective, every state-of-affairs is ineffable and hence has the value \( e \) (in agreement with Priest, 2018). Viewed from the conventional perspective, on the other hand, each state-of-affairs is effable and may have one of the classical values \( t \) and \( f \) (this is where we differ from Priest, 2018). More formally, a model \( v \) (which we shall call a “plurivalent Ke-model”\(^{38} \)) is a function that assigns to each atomic state-of-affairs a one-membered or two-membered subset (a “value-set”, so to speak) of \( \{t, f, e\} \) that includes \( e \) and that satisfies the following conditions (where an underlined formula stands for a state-of-affairs): \(^{39} \)

1. \( f \) belongs to \( v(\neg A) \) if \( t \) belongs to \( v(A) \) and \( t \) belongs to \( v(\neg A) \) if \( f \) belongs to \( v(A) \), otherwise \( v(\neg A) = v(A) \);
2. \( v(A \& B) = \{e\} \cup \{t\} \) if \( t \) belongs to both \( v(A) \) and \( v(B) \), \( v(A \& B) = \{e\} \) if \( v(A) \) or \( v(B) = \{e\} \), otherwise \( v(A \& B) = \{e, f\} \);
3. \( v(A \lor B) = \{e\} \cup \{f\} \) if \( f \) belongs to both \( v(A) \) and \( v(B) \), \( v(A \lor B) = \{e\} \) if \( v(A) \) or \( v(B) = \{e\} \), otherwise \( v(A \lor B) = \{e, t\} \). \(^{40} \)

\(^{37} \) The assertion that “[v]iewed from the ultimate perspective, every state-of-affair is ineffable and hence has the value \( e \)” in the meta-language should not be regarded as violating Madhyamaka’s doctrine of ineffable or Madhyamaka’s practice. One reason is that such an assertion is still made from the conventional perspective. The main reason, however, is that the model-theoretical semantics is formulated in the meta-language; such meta-talks, including the model itself and all kinds of assertions that can be made in or about it, are only heuristic tools for helping us to decide which sentences in the object-language can be asserted, and, if so, from which perspective. Since every state-of-affair has the ultimate value \( e \) in a model, this indicates that nothing can really be truly asserted about any state-of-affair from the ultimate perspective. However, sentences can still be asserted from the conventional perspective. If \( A \)’s value-set contains \( t \), then the sentence “\( A \)” is assertable from the conventional perspective. Similarly, if \( A \)’s value-set contains \( f \), then the sentence “\( \neg A \)” is assertable from the conventional perspective.

\(^{38} \) Plurivalent semantics allows a sentence to have more than one value in a model. The semantic we will give is obviously a plurivalent one. It is called “Ke” because it is essentially a weak Kleen semantics \( K \) with one more value \( e \).

\(^{39} \) In terms of truth tables, the three semantic rules can be explained by the following tables:

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<thead>
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<th>( A )</th>
<th>( \neg A )</th>
<th>&amp;</th>
<th>{e, t}</th>
<th>{e}</th>
<th>{e, f}</th>
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<td>{e, t}</td>
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</table>

\(^{40} \) It may be questioned why we do not have a connective or an operator, corresponding to the ineffable value \( e \) or the non-committal denial (or rejection), in our language. The main
The main ideas behind the model and the value-conditions of various state-of-affairs are actually quite simple: each state-of-affair has an “ultimate” value $e$ and perhaps an extra classical “conventional” value within its value-set. The value $e$ is “infectious” in the sense that any state-of-affair that has a part with $\{e\}$ as its value-set will also have the value $\{e\}$. The value-set for a complex state-of-affairs with no part with the value-set $\{e\}$ is then determined in the ordinary classical way. Since the value $e$ stands for “ineffable”, the stipulation that each state-of-affairs has at least the value $e$ means that each state-of-affairs is ineffable from the ultimate perspective. Yet, each state-of-affairs may also have a classical truth value $t$ or $f$, which means that each state-of-affairs may also have an effable aspect and is either true or false when viewed from the conventional perspective. This formal regimentation seems to be quite faithful to Nāgārjuna’s two-truth theory. As usual, we define an argument to be valid if it preserves the designated value $t$ in every plurivalent Ke-model. It can be proved that the resultant logic, while much weaker than the classical logic, still preserves many classical valid inferential rules such as Modus Ponens, Modus Tollens, and Hypothetical Syllogism, but we will not give the proofs here.

How does this semantics verify that the Prāsaṅgika is the right interpretation of how Nāgārjuna rebuts his opponent? Notice that the classical valid rule *reductio ad absurdum* is no more valid in the above plurivalent Ke-semantics (see the Proof below), so that the derivation of an absurdity from the assumption of a state-of-affairs no more guarantees that the negation of it obtains though it still guarantees that the state-of-affairs must not obtain (and hence can be “deleted”). Because of the importance of this conclusive claim, we give its proof here:

Proof: (Here we take reductio ad absurdum as reduction to contradiction, not absurdity in general, for simplicity.) Reduction to contradiction is the rule “if A entails contradiction, then infer $\neg A$”. Now this is invalid in Ke-semantics because while $(p \& \neg p)$ entails $(p \& \neg p)$, $(p \& \neg p)$ may still not be true (consider a valuation $v$ in which $v(p) = \{e\}$). On the other hand, this does not mean that we should accept any contradiction (a state-of-affairs of the form “$p \& \neg p$”) at all or any state-of-affairs that entails a contradiction, because that would require some state-of-affairs to have both $t$ and $f$ in its value-set,

reason is that, as we said in the previous section, the non-committal denial (or rejection) is not a part of our language but a speech act (or a mental attitude). It is, unlike a connective or an operator, unable to be combined with other elements in a language to form a complex whole and therefore should not be taken to be a connective or an operator of our language.
which is impossible for any Ke-model v. Therefore, if A entails a contradiction, A (and \( \neg A \)) can never be true according to Ke-semantics, and hence A (and \( \neg A \)) should be denied (or rejected).

Therefore, even if Mādhyamika successfully draws an absurdity from his opponent’s thesis and therefore shows that the thesis cannot be true (or can be deleted), s/he is not thereby warranted to conclude that the negation of the thesis is true. Indeed, s/he should not conclude this if s/he is to view things from the ultimate perspective. This formal regimentation is in the spirit of the Prāsaṅgika and Nāgārjuna, but not with the Svātantrika. It also explains why prasaṅga, rather than the classical rule reductio ad absurdum, is the right way for a Mādhyamika to rebut his opponent.

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